MATH 31B, LECTURE 1
FINAL EXAMINATION
JUNE 15, 2012

Name:_____

UID:_____

Signature: _____

TA: (circle one) Charles Marshak Theodore Gast Andrew Ruf

Discussion meets: (circle one) Tuesday Thursday

Instructions: The exam is closed-book, closed-notes. Calculators are not permitted. Answer each question in the space provided. If the question is in several parts, carefully label the answer to each part. Do all of your work on the examination paper; scratch paper is not permitted. If you continue a problem on the back of the page, please write "continued on back".

Each problem is worth 10 points.

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Problem 1: (Multiple Choice). Indicate your answer in the provided box.

(i) Evaluate the infinite series: $\displaystyle\sum_{n=0}^{\infty} \frac{1+2^n}{4^{n+1}}$.

(a) 1 (b) 16/3 (c) 5/6 (d) 10/3 (e) 4/3 ☐

(ii) Evaluate the infinite series: $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$.

(a) 3/2 (b) 2 (c) 1/2 (d) 3/4 (e) 1 ☐

(iii) Evaluate the infinite series: $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$.

(a) $\pi/2$ (b) e^{-1} (c) 1/2 (d) $\pi/4$ (e) 3/4 ☐

(iv) Evaluate the limit: $\displaystyle\lim_{n\to\infty} \sqrt[n]{n^2+1}$.

(a) 1/2 (b) $+\infty$ (c) 0 (d) 1 (e) 2 ☐

(v) Which diff. equation does $y = 1 + \dfrac{1\cdot x^3}{3!} + \dfrac{4\cdot 1\cdot x^6}{6!} + \dfrac{7\cdot 4\cdot 1}{9!}x^9 + \dfrac{10\cdot 7\cdot 4\cdot 1}{12!}x^{12} + \cdots$ satisfy?

(a) $y' = x^2 y$ (b) $y'' = xy$ (c) $y'' + xy' - x^2 y = 0$ (d) $y'' = y$ (e) None of these ☐

Problem 2: (Multiple choice). Indicate the Maclaurin series (a-h) corresponding to $f(x)$.

(i) $f(x) = \frac{2x}{2+x^2}$. ☐

(ii) $f(x) = x\ln(1+x)$. ☐

(iii) $f(x) = \cos x^2$. ☐

(iv) $f(x) = \tan^{-1} x$. ☐

(v) $f(x) = \sin x \cos x$. ☐

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n}}{(2n)!}$ (b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{2n}$ (c) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{(2n+1)}}{(2n+1)!}$ (d) $\displaystyle\sum_{n=2}^{\infty}(-1)^n \frac{x^n}{n-1}$

(e) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{4^n \cdot x^{2n+1}}{(2n+1)!}$ (f) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n+2}}{(2n+1)!}$ (g) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ (h) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2^n}$

Extra scratch paper for Problems 1 and 2

Problem 3: Evaluate the limits.

(a) $\lim_{x\to 1}\left(\dfrac{1}{\ln x}+\dfrac{1}{1-x}\right)$

(b) $\lim_{x\to\infty}\left(\dfrac{x}{x+2}\right)^{x}$

Problem 4: Let $f(x) = (\cos x)^{\tan x}$, find $f'(x)$.

Problem 5:

(a) Evaluate the indefinite integral $\int \sin^3(x)\, dx$.

(b) Evaluate $\lim_{R\to\infty} \int_{-R}^{R} \sin^3(x)\, dx$.

(c) Evaluate the improper integral $\int_{-\infty}^{\infty} \sin^3(x)\, dx$, or show that it diverges.

Problem 6: Evaluate the integral: $\displaystyle\int_{-\infty}^{0} e^x \sqrt{1 - e^{2x}}\, dx$.

Problem 7:

(a) Find the partial fractions decomposition of $\dfrac{2}{x^3 - 2x^2 + 2x}$.

(b) Evaluate the indefinite integral: $\displaystyle\int \frac{2}{x^3 - 2x^2 + 2x}\, dx$.

Problem 8: Determine whether the given series converges absolutely, converges conditionally or diverges. You must carefully justify your answer.

(a) $\displaystyle\sum_{n=2}^{\infty}(-1)^n \sin\left(\frac{\pi}{n}\right)$.

(b) $\displaystyle\sum_{n=2}^{\infty}(-1)^n \frac{n^2}{4^n - 2^n - n + 1}$.

Problem 9: Determine the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{2^n + n^2}$.

Problem 10: Let $f(x) = (1 + x^2) \tan^{-1}(x)$.
(a) Find the Maclaurin series for $f(x)$.
(b) Find $f^{(9)}(0)$.